Tutorial Notes on

CHAOS IN POWER ELECTRONICS

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Chaos in Power Electronics

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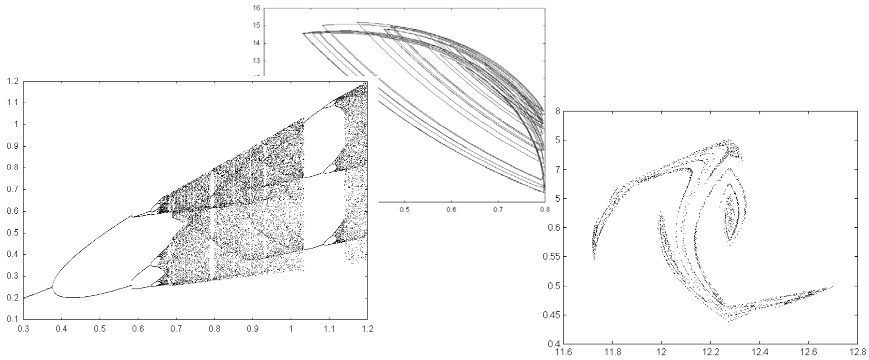
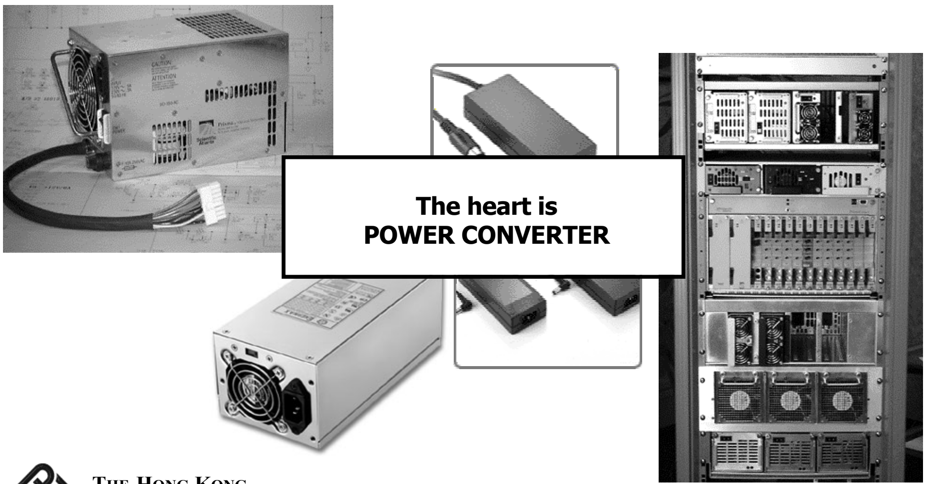


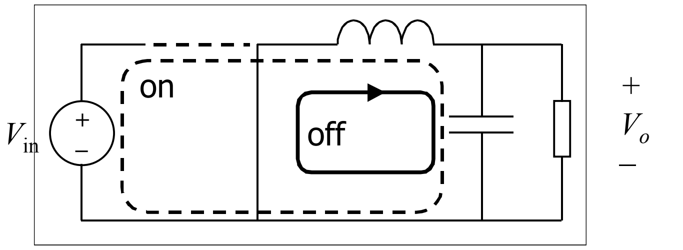
What are power electronics circuits?



The heart is
POWER CONVERTER

Simple dc/dc converters

e.g., buck (step-down) converter

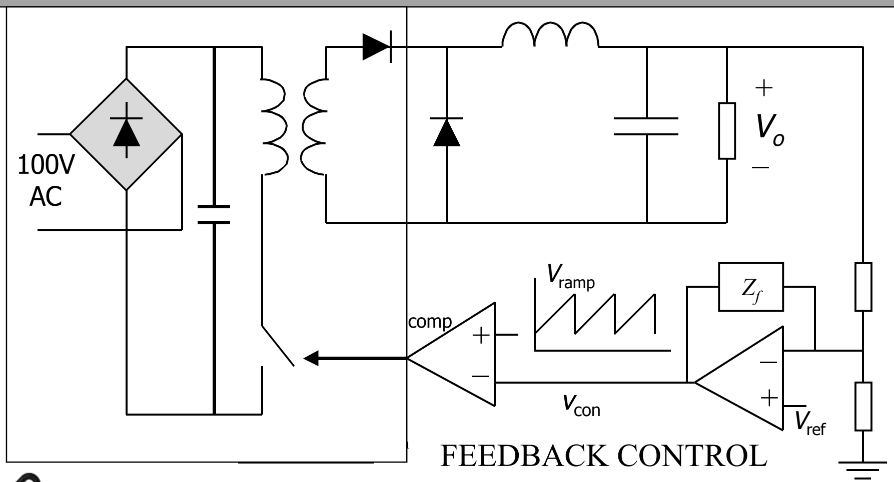


- The switch is turned on and off at high frequency.
- The output is equal to

$$V_o = DV_{in}$$

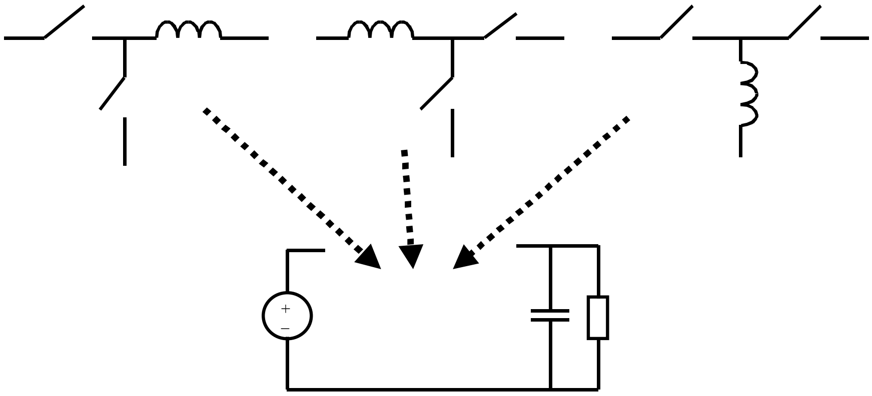
$$D = \text{duty cycle} = \frac{\text{on time}}{\text{period}}$$

Power supply AC/DC adaptor



Basic converters

Buck (step-down) Boost (step-up) Buck-boost (step-up-down)



Nature of operation

Time varying — different systems at different times

AND

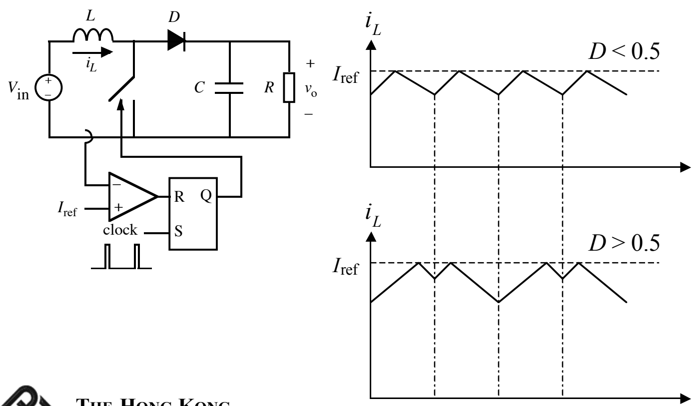
Nonlinear — the time durations are related nonlinearly with the output function

Nonlinearity – the default property of power electronics circuits

- Power electronics engineers/researchers are dealing with nonlinear problems
- Much of power electronics is about identifying nonlinear phenomena and "taming" them to do useful applications
- Classic examples:
  - Averaging (R. David Middlebrook, Richard Bass)
  - Discrete-time modeling (Harry Owen, Fred Lee)
  - Stability analysis (George Verghese)
  - Phase-plane analysis/control (Fred Lee, Ramesh Oruganti)
  - Series approximation (Richard Tymerski)

Classic example of nonlinear study

Boost converter with current-mode control



Simple analysis reveals a change of stability status at a critical duty cycle of 0.5.

The circuit is actually 'stable' beyond the critical point, though operates with a longer period.

This **period-doubling** phenomenon was observed long ago.

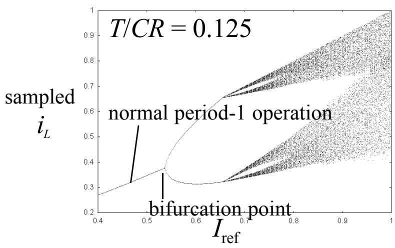
Recent study from a bifurcation perspective

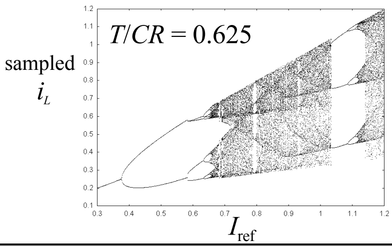
With the help of computers, we can study the phenomenon in more detail.

Bifurcation diagrams (panaromic view of stability status)

> Sampled variable at steady state versus parameter, e.g., $i_L(nT)$ vs. I_{ref}

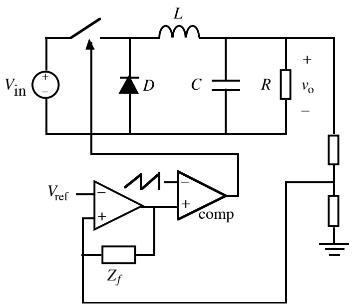
We can plot bifurcation diagrams for different sets of parameters

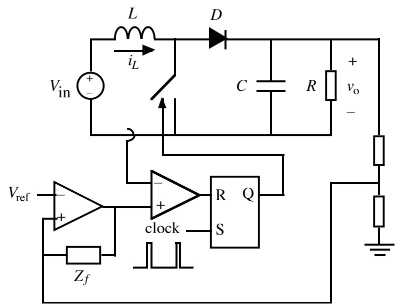




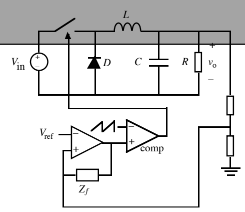
Circuits whose bifurcation behaviors have been studied in detail

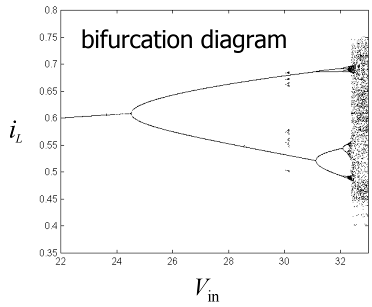
Voltage-mode and current-mode controlled simple buck and boost converters, and many others

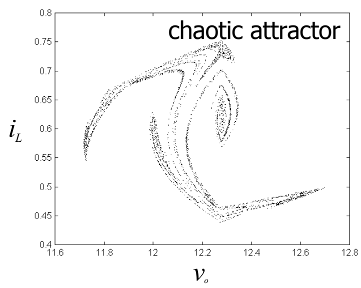




Period-doubling and chaos in voltage-mode controlled buck converter

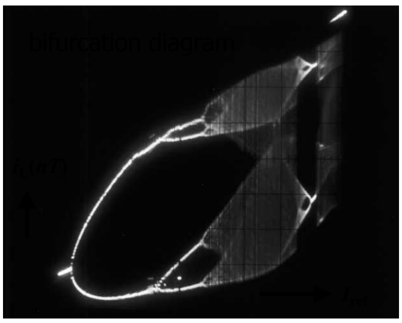


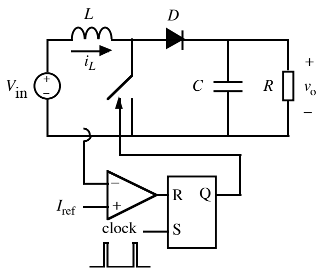




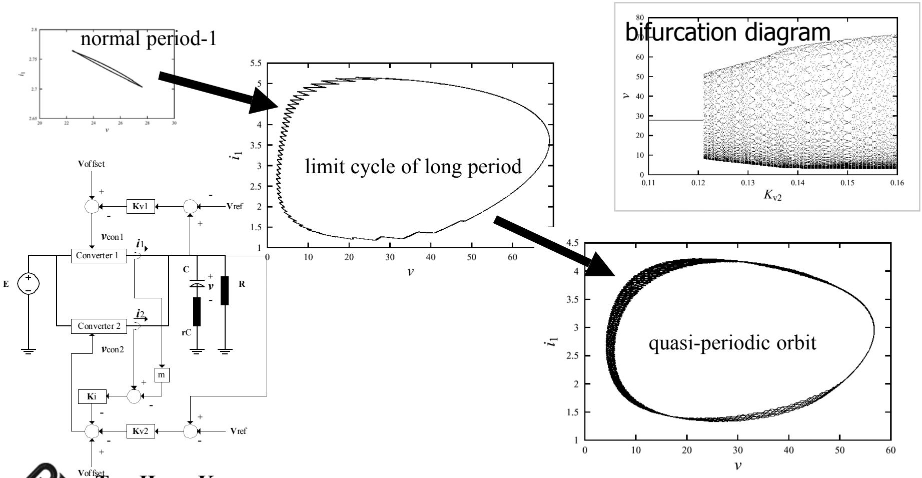
Hamill *et al.* (1990)

Period-doubling and chaos in current-mode controlled boost converter

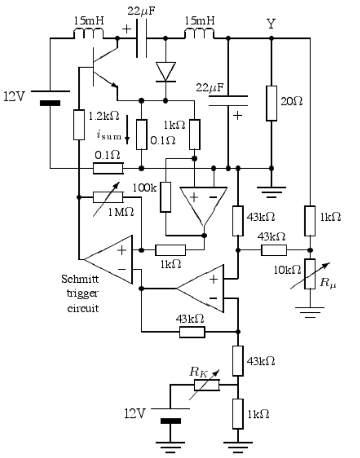


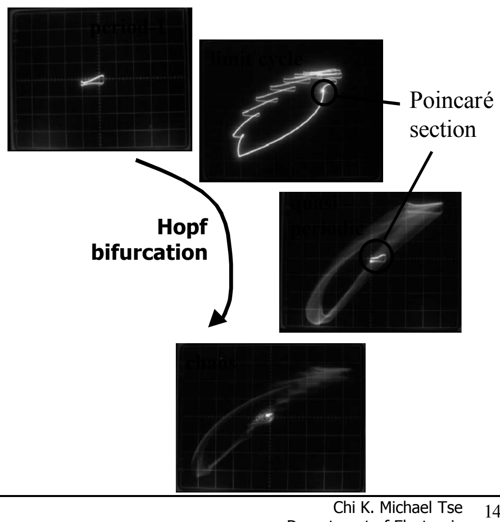


Hopf Bifurcation in parallel boost converters



Hopf Bifurcation in free-running Ćuk converter





Literature review

- Hamill and Jefferies [1988] - First analysis of bifurcation and chaotic dynamics in a first order PWM voltage-mode controlled converter.
- Deane and Hamill [1990] - Analysis of bifurcation in first order and second order PWM buck converters.
- Hamill *et al*. [1992] - Derivation of an iterative map to analyze bifurcation in a buck converter in continuous mode.
- Deane [1992] - First report on chaotic behaviour in a current-controlled boost converter.
- Tse [1994] - Derivation an iterative map to demonstrate period-doubling cascades in a boost converter in discontinuous mode.

Literature review (cont.)

- Chakrabarty *et al*. [1995] / Fossas and Olivar [1996] - Further study of chaos in a PWM buck converter.
- Dobson [1995] - Study of bifurcation in thyristor and diode circuits
- Poddar *et al*. [1995] / Batlle *et al*. [1996] - Control of chaos in dc/dc converters.
- Tse and Chan [1995] - Study of bifurcation and chaos in a fourth order current-controlled Cuk converter.
- Banerjee *et al.* [1997] - Analysis of coexisting attractors in buck converter
- Chan and Tse [1996] - Study of bifurcation in current-mode converter with and without feedback
- Banerjee *et al*. [1997] - Examination of current-mode converters in the light of "border collision bifurcation".

Literature review (cont.)

- Tse [1997] – Analysis of autonomous Cuk converters using averaged models.
- Di Bernardo *et al*. [1998] - Study of various sampling and their applications in the identification of bifurcation and chaos.
- Chan and Tse [1998] – Proof of period-doubling in discontinuous converters using Schwarzian derivatives
- Di Bernardo *et al*. [1998] - Analysis of the non-smooth dynamics (such as grazing, skipping and sliding) of dc/dc converters.
- El Aroudi *et al*. [1999] - Identification of quasi-periodicity and chaos in a boost converter.
- Mazumder, Nayfeh and Borojevich [1999] - Fast- and slows-scale instabilities.
- Iu and Tse [2000] - Study of bifurcation in parallel converters
- Orabi and Ninomiya [2002] / Tse [2002] - Analysis of power factor correction converters

Literature review (cont.)

Review articles

D.C. Hamill, *Proc. NDES'95,* pp. 165-177. 1995.

C.K. Tse, *CAS Newsletter,* pp. 14-48, March 2000.

I. Nagy, *Automatica* **42**, pp. 117-132, 2001.

S. Banerjee et al., Ch. 1, *Nonlinear Phenomena in PE,* IEEE Press 2001.

C.K. Tse and M. di Bernardo, *IEEE Proceedings* **90**, pp. 768–781, 2002.

Current state of findings

- **Two types of** ***bifurcation**** **seen in power electronics**
- **Standard bifurcations** (found in other systems as well)
  - Period-doubling
  - Hopf (Neimark-Sacker)
  - Saddle-node
- **Border collision** (characteristic of power electronics)
  - Abrupt change of behavior due to a *structural change*

*Bifurcation refers to sudden change of qualitative behaviour of a dynamical system when a certain parameter is varied.

Current state of findings

Who will get what?

- **Standard bifurcations**
  - Buck converters (voltage-mode) – period-doubling
  - Boost converters (voltage-mode) – Hopf
  - Dc/dc converters in DCM – period-doubling
  - Most dc/dc converters (current-mode) – period-doubling
  - Other types – variety: saddle-node, crisis, etc.
- **Border collision** (characteristic of power electronics)
  - All standard bifurcations are interrupted by border collision

Comparisons

- **Standard bifurcations**
  - Loss of stability
  - No structural change
  - Standard appearance bifurcation diagrams
- **Border collision**
  - Loss of "operation"
  - Structural change
  - Non-standard appearance in bifurcation diagrams, e.g., bendings, jumps, etc

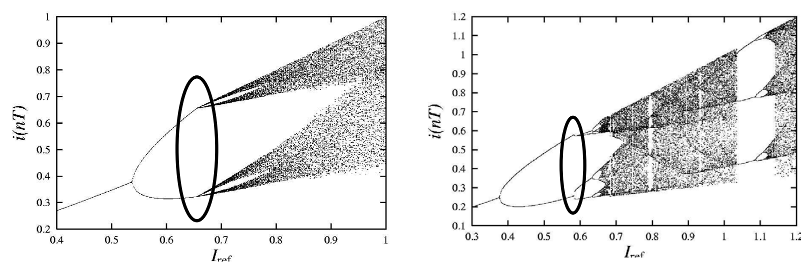
Structural change in switching converters
= Alteration in topological sequence

e.g., change of operating mode, reaching a saturation boundary

Border collision – *signature bifurcation* of power electronics circuits

Border collision

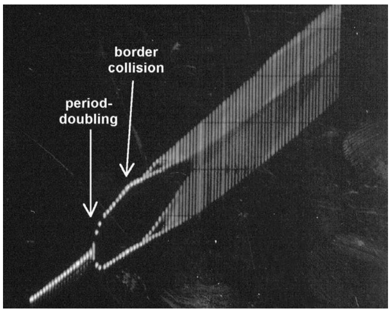
Non-smooth phenomena "always" observed in power electronics circuits



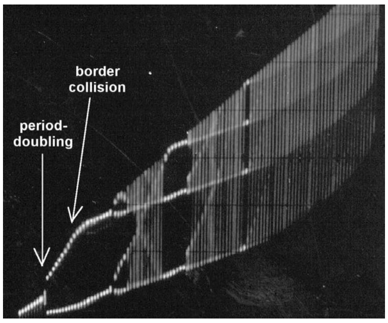
boost converter under current-mode control

Border collision

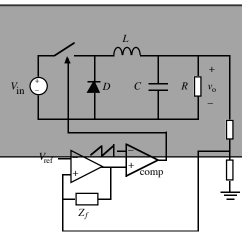
Essence: standard bifurcation being interrupted



boost converter under current-mode control

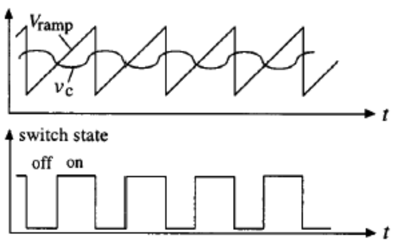


Border collision – a practical viewpoint

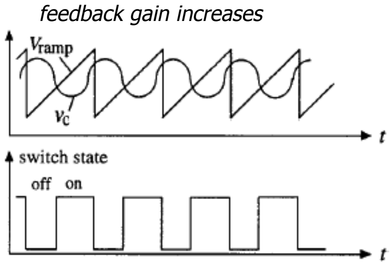


Excessive swing of control signal
Duty cycle saturation
Out-of-range operation, preventing continuation of standard bifurcations

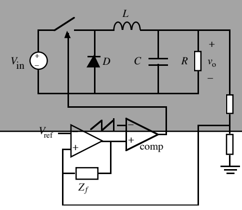
e.g., buck converter in voltage-mode control



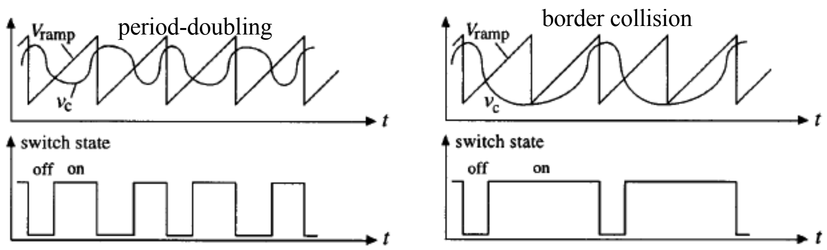
feedback gain increases



Border collision – a practical viewpoint



Further increase in feedback gain…



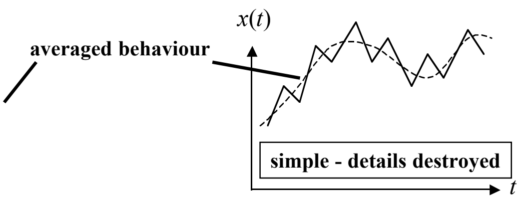
Basic problems in analysis

- **What models to use?**
- **What techniques to use?**
- *An open question* **:**
  - **What are the applications?**
  - **(What do we use the results for?)**

What models to use?

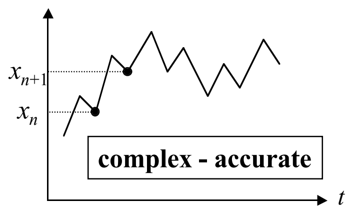
- Averaged models

$$\frac{dx}{dt} = f(x, \mu, t)$$



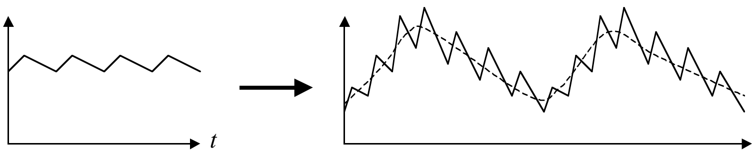
- Discrete-time models

$$x_{n+1} = F(x_n, \mu)$$

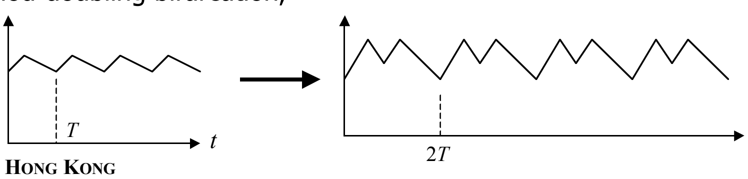


What models to use?
Clue: Typical nonlinear behavior observed in dc/dc converters

- Standard bifurcations
  - Hopf bifurcation,

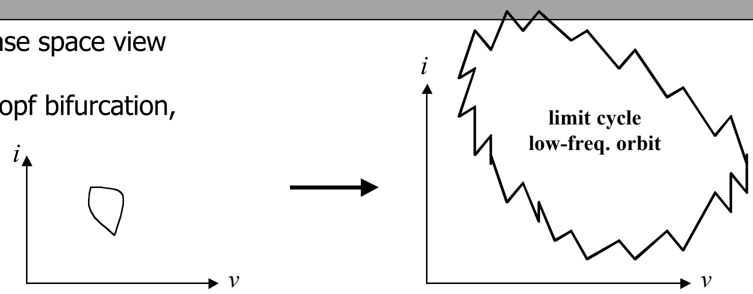


  - Period-doubling bifurcation,

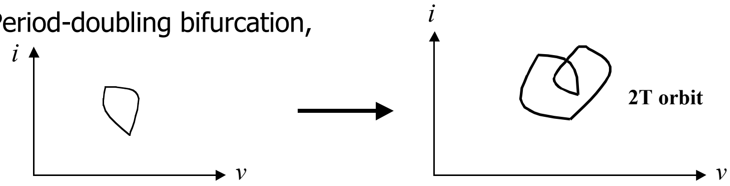


What models to use?
Clue: Typical nonlinear behavior observed in dc/dc converters

- Phase space view
  - Hopf bifurcation,

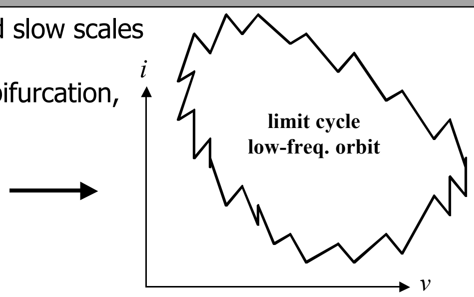


  - Period-doubling bifurcation,



What models to use?
Fast and slow scale dynamics

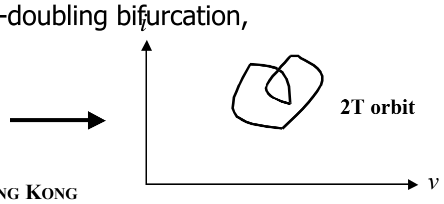
- Fast and slow scales
  - Hopf bifurcation,



low **-frequency phenomenon**

– averaged models are capable

  - Period-doubling bifurcation,



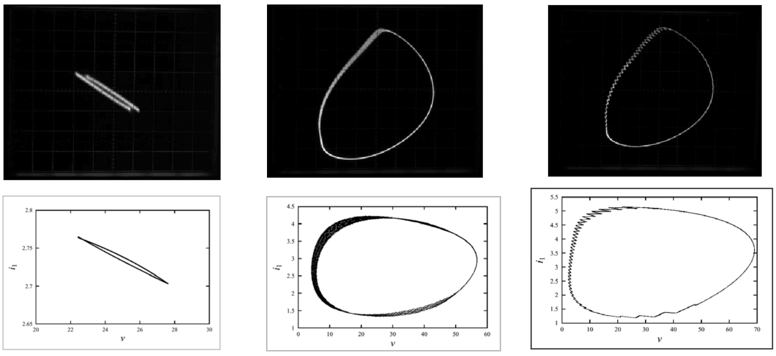
high **-frequency phenomenon**

– discrete-time models are needed

What models to use?
Clue: Experiments and computer simulations

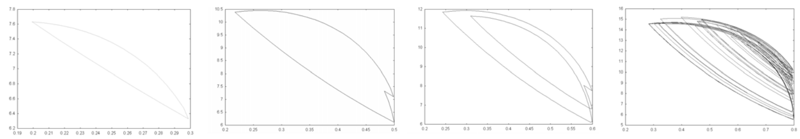
e.g., In parallel boost converters, as feedback gain increases, we observe a series of changes…

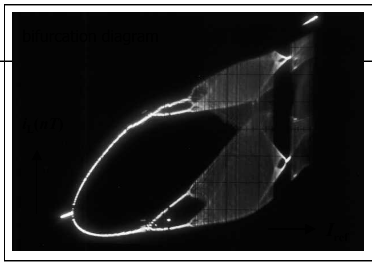
Stable period-1 ⟶ quasi-periodic ⟶ limit cycle



What models to use?
Clue: Experiments and computer simulations

e.g., In a current-mode boost converter, we see period-doubling:





What models to use?
Clue: Experiments and computer simulations

EXAMPLES:

For the parallel boost converters under master-slave control, we see a slow-scale phenomenon.

Thus, averaged models should be adequate!
(Iu and Tse, ISCAS'2002)

For the current-mode controlled boost converter, we see fast-scale phenomenon.

Thus, we must resort to discrete-time models.
(Chan and Tse, IEEE TCAS1 1997)

Analysis techniques

The basic questions of practical importance are

Where and when it happens?
Location of boundary of operation – bifurcation point

How it happens?
Identification of the type of bifurcation

Technique – Averaging approach

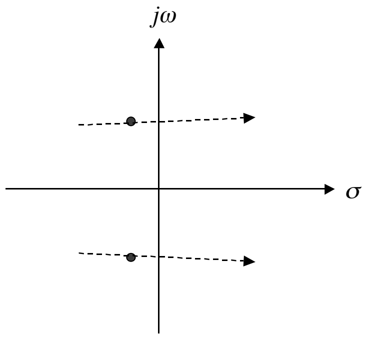
Averaging approach

- Derive a set of continuous averaged equations:

$$\dot{x} = f(x)$$

- Examine the Jacobian, $J(X_Q)$ and find the loci of the eigenvalues when a bifurcation parameter is varied.

- Identify the condition for the eigenvalue(s) moving across the imaginary axis in the complex plane:
  - *e.g., a pair of complex eigenvalues moving across the imaginary axis implies a Hopf bifurcation.*



Technique – Averaging approach

Advantages:

- Widely used and well known.

- Relatively easy to derive the continuous averaged equation.

Limitation:

- Only able to predict low-frequency slow-scale bifurcation behaviour such as Hopf bifurcation.

Technique – Discrete-time map approach

Discrete-time map approach

• Derive a discrete time map (iterative function *f*):

$$x_{n+1} = f(x_n, d)$$

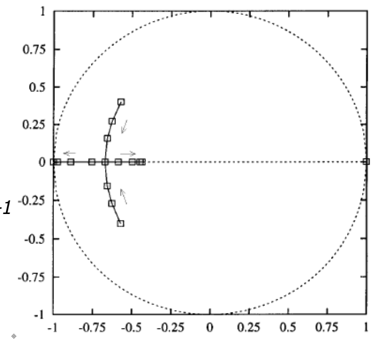
• Examine the Jacobian, $J(X_Q)$ and find the loci of the eigenvalues when a bifurcation parameter is varied.

• Identify the condition for the eigenvalue(s) moving out the unit circle in the complex plane:

$$\det[\lambda I - J(X_Q)] = 0$$

•

•*e.g., one of the eigenvalues moving out through -1 the implies a period-doubling.*



Technique – Discrete-time map approach

Advantages:

• Provide a relatively complete behavioral information.

• Able to predict standard bifurcations such as period-doubling bifurcation, Hopf bifurcation and saddle-node bifurcation.

Limitation:

• Derivation of the iterative map is more complicated compared to the continuous-time averaged equation.

Analysis of border collision

Basic principle:

It usually involves *change of the form of the qualitative model* before and after the collision.

Results (reported in Physics literature):

There are theoretical publications (C. Grebogi, H.E. Nusse, S. Banerjee, M. di Bernardo) on the type of transition at the collision. These transitions are usually "sudden" changes, e.g.,

from period-1 to chaos
from period-1 to another period-1

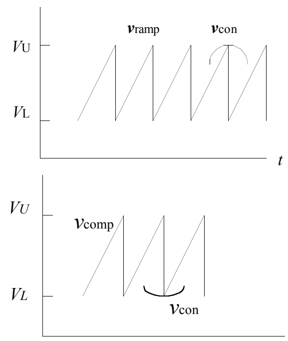
Analysis of border collision

Of engineering interest is the location of the border.

Analysis involves identifying the mechanism that causes the collision,

e.g., control signal swings to far up or down

One can analyze the situation and derive formula for the critical point.



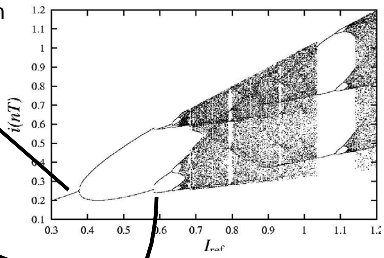
Summary of observations

Standard bifurcations such as period-doubling and Hopf bifurcation are commonly observed in dc/dc converters.

— practically more important because standard bifurcation is always the first bifurcation next to the usual stable operation

Border collision bifurcation comes into play to disrupt the growth of standard bifurcations.

— signature phenomenon in power electronics



Possible engineering applications

Prediction / Better understanding
Systematic collection of results concerning bifurcation to form a useful design guide.

operation boundary
instability features —
[e.g., recent finding on PFC converter]

Design
Use of chaotic operation to advantage.

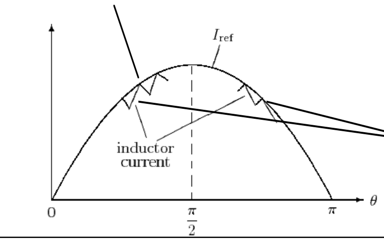
transient speed ?
EMC ?

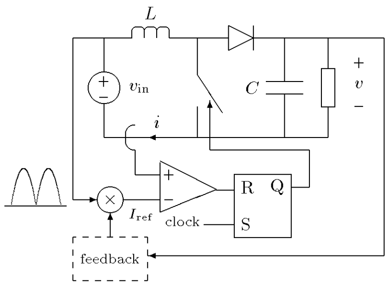
Application example
– bifurcation analysis in PFC boost converter

PFC boost rectifier

= typically a boost converter under current-programming control

The input current is forced to track the input voltage waveshape





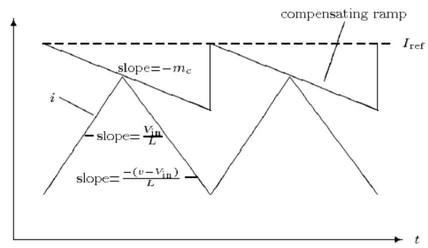
Note: essentially slope compensation

Application example
– bifurcation analysis in PFC boost converter

We observe asymmetrical slope compensation in this PFC boost rectifier:

+ve slope compensation in [0, π/2]
--> less stable
–ve slope compensation in [π/2, π]
--> more stable

Bifurcation analysis can reveal interesting phenomenon…



Discrete-time map:

$$i_{n+1} = \left(\frac{M_c + 1 - v/V_{in}}{M_c + 1}\right) i_n + \text{higher order terms}$$

where $M_c = \dfrac{m_c L}{v_{in}}$ just for convenience.

Application example
– bifurcation analysis in PFC boost converter

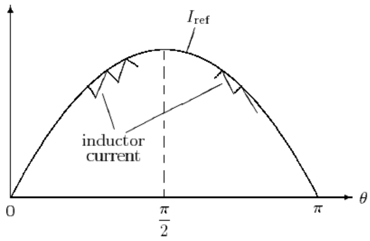
Bifurcation analysis

Suppose the input voltage is

$$v_{\rm in}(\theta) = \hat{V}_{\rm in}\,|\sin\theta|$$

Relating the compensation slope m_c with the input voltage variation for the PFC case:

$$m_c = -\frac{di_{\rm ref}}{dt} \approx \omega_m \hat{I}_{\rm L}\cos\theta$$



Question:

Would there be fast-scale instability (e.g., period-doubling)? At what phase angle would it occur?

Application example
– bifurcation analysis in PFC boost converter

Bifurcation analysis

Jacobian:

$$\delta i_{n+1} = \left(\frac{M_c}{1+M_c} - \frac{D}{(1-D)(1+M_c)}\right)\delta i_n + O(\delta i_n^2).$$

Characteristic multiplier:

$$\lambda = \frac{M_c}{1+M_c} - \frac{D}{(1-D)(1+M_c)}.$$

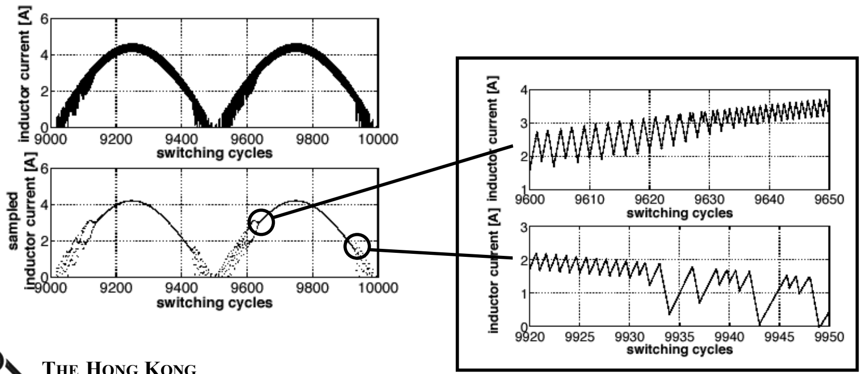
Period-doubling occurs when $\lambda = -1$. →

Critical phase angle (algebra omitted):

$$\theta_c = 2\arctan\left(\frac{2\hat{V}_{\rm in} \pm \sqrt{4\hat{V}_{\rm in}^2 - v^2 + 4\omega_m^2 \hat{I}_{\rm in}^2 L^2}}{v - 2\omega_m \hat{I}_{\rm in} L}\right)$$

Application example
– bifurcation analysis in PFC boost converter

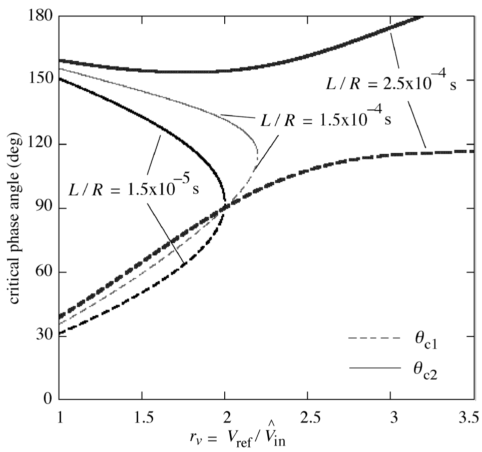
Fast scale instability at phase angle $\theta_c = 2\arctan\left(\frac{2\hat{V}_{in} \pm \sqrt{4\hat{V}_{in}^2 - v^2 + 4\omega_m^2 \hat{I}_{in}^2 L^2}}{v - 2\omega_m \hat{I}_{in} L}\right)$



Application example
– bifurcation analysis in PFC boost converter

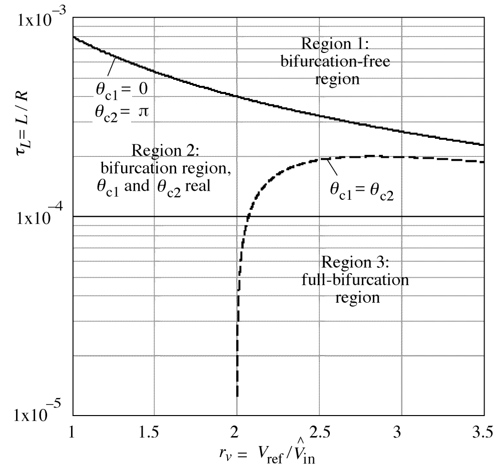
This formula allows us to do a number of things:

e.g.,
Predicting fast-scale instability

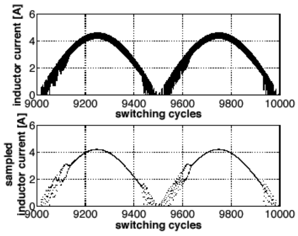


Application example
– bifurcation analysis in PFC boost converter

Defining stability boundaries



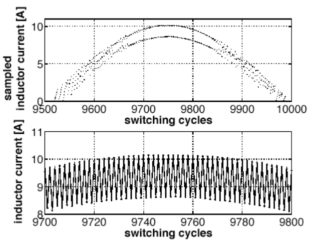
Application example
– bifurcation analysis in PFC boost converter

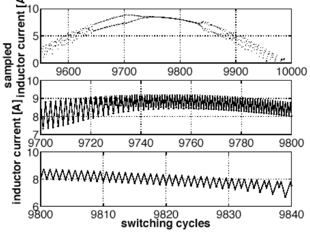


$0 < \theta_{c1} < \frac{\pi}{2}$

$\frac{\pi}{2} < \theta_{c2} < \pi$

region 2 ⟶ region 3





$\theta_{c1} > \frac{\pi}{2}$

$\theta_{c2} < \frac{\pi}{2}$

$\theta_{c1} = \frac{\pi}{2}$

$\frac{\pi}{2} < \theta_{c2} < \pi$

Conclusion

Recap:
- Power electronics engineers/researchers are dealing with nonlinear problems
- Much of power electronics is about identifying nonlinear phenomena and "taming" them to do useful applications

For power electronics (nonlinear systems in general),

– "stability" refers to operation in the expected regime
– a variety of ways the system can become unstable (to get away from the usual operation)
– a number of affecting parameters

— **BIFURCATION ANALYSIS**

Future work

Engineers frequently ask:

What are the applications of chaos and bifurcation studies?

Topics of future research:

- Reorganizing results in terms of practical operating conditions and parameters
- Developing design-oriented bifurcation procedures
- Identifying new phenomena in practical circuits

To probe further

Books

- C.K. Tse, *Complex Behavior of Switching Power Converters,* Boca Raton: CRC Press, 2003.
- S. Banerjee and G.C. Verghese (Eds.), *Nonlinear Phenomena in Power Electronics: Attractors, Bifurcations and Nonlinear Control,* New York, IEEE Press, April 2001.

Review paper

- C.K. Tse and M. di Bernardo, "Complex behavior of switching power converters," *Proceedings of the IEEE,* vol. 90, no. 5, pp. 768–781, 2002.

Journals

- IEEE Transactions on Circuits and Systems Part I
- International Journal of Bifurcation and Chaos
- International Journal of Circuit Theory and Applications

Our research group homepage: **http://chaos.eie.polyu.edu.hk**

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