TABLAS

TRANSFORMADA DE FOURIER

PROPIEDADES		
DEFINICION	$f(t)$	$F(\omega)=\int_{-\infty}^{+\infty} f(t)\, e^{-j\omega t} dt$
DUALIDAD	F(t)	$2\pi\, f(-\omega)$
ESCALA	$f(at)$	$\frac{1}{\lvert a\rvert} F\left(\frac{\omega}{a}\right)$
DESPLAZAMIENTO EN EL TIEMPO	$f(t-t_0)$	$e^{-j\omega t_0} F(\omega)$
DESPLAZAMIENTO EN FRECUENCIA	$e^{j\omega_0 t}\ f(t)$	$F(\omega-\omega_0)$
DERIVACION EN EL TIEMPO	$f^{(n)}(t)\ , n\in N$	$(j\omega)^n\ F(\omega)$
DERIVACION EN FRECUENCIA	$(-jt)^n\, f(t)\ , n\in N$	$F^{(n)}(\omega)$
INTEGRACION EN EL TIEMPO	$\int_{-\infty}^{t} f(u)du$	$\frac{1}{j\omega} F(\omega)+\pi F(0)\ \ \delta(\omega)$
CONVOLUCION EN EL TIEMPO	$f(t) * g(t)$	$F(\omega)G(\omega)$
CONVOLUCION EN FRECUENCIA	$f(t)\ g(t)$	$\frac{1}{2\pi} F(\omega) * G(\omega)$
	$f(t)\cos(\omega_0 t)$	$\frac{1}{2} F(\omega-\omega_0)+\frac{1}{2} F(\omega+\omega_0)$
	$f(t)\,\mathrm{sen}(\omega_0 t)$	$\frac{1}{2j} F(\omega-\omega_0)-\frac{1}{2j} F(\omega+\omega_0)$
Relación de Parseval: $E=\int_{-\infty}^{\infty}\lvert f(t)\rvert^2\ dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert F(\omega)\rvert^2 d\omega$		

PARES TRANSFORMADOS	
FUNCIÓN	**TRANSFORMADA**
$e^{-at}\, u_s(t)\ ,\ a>0$	$\frac{1}{j\omega + a}$
$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$
$e^{-at^2}\ ,\quad a \neq 0$	$\sqrt{\frac{\pi}{a}}\ e^{-\omega^2/4a}$
$P_a(t)$	$a\ \mathrm{sinc}\left(\frac{a\omega}{2}\right)$
$T_{2b}(t)$	$b\,\mathrm{sinc}^2\left(\frac{b\omega}{2}\right)$
$\frac{t^{n-1}}{(n-1)!} e^{-at}\ u_s(t), n \in N\ , a>0$	$\frac{1}{(j\omega + a)^n}$
e^{-at} sen bt $u_s(t),\ a>0$	$\frac{b}{(j\omega+a)^2+b^2}$
e^{-at} cos bt $u_s(t)\ ,a>0$	$\frac{j\omega+a}{(j\omega+a)^2+b^2}$
$\delta(t)$	1
$\delta^{(n)}(t),\ n \in N$	$(j\omega)^n$
$u(t)$	$\pi\,\delta(\omega)+\frac{1}{j\omega}$
$t^n\ ,\ n \in N$	$2\pi\, j^n \delta^{(n)}(\omega)$
$e^{j\omega_0 t}$	$2\pi\,\delta(\omega-\omega_0)$
$\cos\ \omega_0 t$	$\pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$
$\mathrm{sen}\ \omega_0 t$	$-j\pi\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$
$\mathrm{sen}\,\omega_0 t\ \ u_s(t)$	$\frac{\omega_0}{\omega_0{}^2-\omega^2}+\frac{\pi}{2j}\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$
$\cos\ \omega_0 t\ \ u_s(t)$	$\frac{j\omega}{\omega_0{}^2-\omega^2}+\frac{\pi}{2}\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$
$t\, u_s(t)$	$j\pi\,\delta'(\omega)-\frac{1}{\omega^2}$

TRANSFORMADA DE LAPLACE

FUNCIÓN	TRANSFORMADA
$f(t)\,,t\geq 0$	$\mathcal{L}\{f(t)\}=F(s)=\int_0^\infty e^{-st}f(t)dt$
$\alpha f(t)+\beta g(t)$	$\alpha F(s)+\beta G(s)$
1	$1/s$
$\delta(t)$	1
e^{at}	$\frac{1}{s-a}\quad Re\{s\}>Re\{a\}$
t^n	$\frac{n!}{s^{n+1}}\quad Re\{s\}>0$
sen(at)	$\frac{a}{s^2+a^2}$
cos(at)	$\frac{s}{s^2+a^2}$
$e^{at}f(t)$	$F(s-a)$, si $F(s)=\mathcal{L}\{f(t)\}$
$e^{at}t^n$	$\frac{n!}{(s-a)^{n+1}}\,,\; n=1,2,...$
$t^n f(t)$	$(-1)^n\frac{d^n}{ds^n}F(s),\;\; n=1,2,3,...$
$f^{(n)}(t),\;\; n=1,2,3,...$	$s^nF(s)-s^{n-1}f(0^+)-s^{n-2}f'(0^+)-.....-f^{(n-1)}(0^+)$
$\int_0^t f(\tau)d\tau$	$\frac{1}{s}F(s)$
$\int_{-\infty}^t f(\tau)d\tau$	$\frac{1}{s}F(s)-\frac{1}{s}\int_{-\infty}^0 f(t)dt$
$\frac{f(t)}{t}$, $\;si\;\lim_{t\to 0}\frac{f(t)}{t}\;\;EXISTE$	$\int_s^\infty F(u)du$
$f(at),\; a>0$	$\frac{1}{a}F(s/a)$
$f(t-a)u_s(t-a)$	$e^{-as}F(s)$

FUNCIÓN	TRANSFORMADA
$\int_0^t f(v)\,g(t-v)\,dv$	$F(s)\ G(s)$
$f(t) = f(t+T)$	$\frac{1}{1-e^{-sT}}\int_0^T f(t)\,e^{-st}dt$
senh(at)	$\frac{a}{s^2-a^2}$
cosh (at)	$\frac{s}{s^2-a^2}$
e^{bt} senh (at)	$\frac{a}{(s-b)^2-a^2}$
e^{bt} cosh(at)	$\frac{s-b}{(s-b)^2-a^2}$
e^{bt} sen (at)	$\frac{a}{(s-b)^2+a^2}$
e^{bt} cos(at)	$\frac{s-b}{(s-b)^2+a^2}$

T. del valor final

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

T. del valor inicial

$$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$$

VARIABLES DE ESTADO

$$\begin{cases} x(t) = \phi(t - t_0)\ x(t_0) + \int_{t_0}^{t} \phi(t-\lambda)\ \ B\ \ u(\lambda)\ d\lambda \\ y(t) = C\ x(t) + D\ u(t) \end{cases}$$

2ª Forma Canónica

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix} \quad ; \qquad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} ; \qquad D = b_n$$

$$C = \left[(b_0 - a_0\, b_n)\ (b_1 - a_1\, b_n) \dots \dots (b_{n-2} - a_{n-2}\, b_n)\ (b_{n-1} - a_{n-1}\, b_n) \right]$$

Transformación

$$A_v = P^{-1} A P \ ; \ B_v = P^{-1} B \ ; \ C_v = C\,P \ ; \ D_v = D$$

Método de Cayley – Hamilton : $e^{\mathbf{A}t} = \sum_{i=0}^{n-1} \gamma_i(t)\ \mathbf{A}^i$ **,**

Los $\gamma_i(t)$ se obtienen de $e^{\lambda_j t} = \gamma_0(t) + \sum_{i=1}^{N-1} \gamma_i(t)\ (\lambda_j)^i$, $j = 1,2,\dots,N$

Señales de potencia y energía

Para calcular la energía: $\mathrm{E} = \lim_{L\to\infty} \int_{-L}^{L} |f(t)|^2\ dt$

Para analizar si es de potencia: $0 < \lim_{L\to\infty} \frac{1}{2L} \int_{-L}^{L} |f(t)|^2\ dt < \infty$

Propiedades Delta de Dirac

1) $\delta(t - t_0) = 0\ \ \forall\ t / t \neq t_0$

2) $\int_{t1}^{t2} \delta(t - t_0)\ dt = 1 \quad si \quad t_1 \leq t_0 \leq t_2$ extensible a intervalos infinitos

3) Si f(t) es continua en $t_1 = t_0 \Rightarrow f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$

4) $\int_a^b \delta(t-t_0)\, g(t)\, dt = \begin{cases} g(t_0) & si\ a < t_0 < b \\ 0 & si\ a > t_0 > b \end{cases} \qquad g(t)\ es\ contínua\ en\ t = t_0\ ;\ a < b$

5) $\int_{t1}^{t2} \delta(\lambda - t\ )\delta(\lambda - t_0)\, d\lambda = \delta(t-t_0), \quad con\ t_1 < t_0 < t_1$

6) $\int_{t1}^{t2} f(t)\, \delta^{(k)}(\lambda - t_0)\ dt = (-1)^k f^{(k)}(t_0), \quad con\ t_1 < t_0 < t_1$

7) $u(t-t_0) = \int_{-\infty}^{t} \delta(\lambda - t_0)\ d\lambda = \begin{cases} 1 & si\ \ t > t_0 \\ 0 & si\ \ t < t_0 \end{cases}$

8) $\int_{-\infty}^{\infty} \delta(at - t_0)\ dt = \frac{1}{|a|} \int_{-\infty}^{\infty} \delta(t - \frac{t_0}{a})\ dt$

9) $\delta(t) = \delta(-t)$

SERIES DE LAURENT

$$\mathbf{f(z) = \sum_{n=0}^{\infty} a_n (z-z_0)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z-z_0)^n}} \qquad \mathbf{(1)}$$

$$\mathbf{a_n = \frac{1}{2\pi j}\int_C \frac{f(z)}{(z-z_0)^{n+1}} dz \quad (n = 0,1,2,........) \quad (2) \qquad b_n = \frac{1}{2\pi j}\int_C \frac{f(z)}{(z-z_0)^{-n+1}} dz \quad (n = 1,2,.........) \quad (3)}$$

Desarrollos en serie más utilizados:

1) Serie binomial:

$$(1+z)^{\alpha} = \sum_{n=0}^{\infty} \binom{\alpha}{n} z^n\ ,\ |z| < 1;\ \text{donde} \binom{\alpha}{0} = 1,\ \binom{\alpha}{n} = \frac{\alpha(\alpha-1)........(\alpha-n+1)}{n!}$$

Casos particulares:

1-1) $\frac{1}{1+z} = \sum_{n=0}^{\infty} (-1)^n z^n,\ |z| < 1\ \ ó\ \ (1+z)^{-1} = 1 - z + z^2 - z^3 + ...\ ....$

1-2) $\frac{1}{1-z} = \sum_{n=0}^{\infty} (z)^n,\ |z| < 1\ \ ó\ \ (1+(-z))^{-1} = 1 + z + z^2 + z^3 + ...\ ....$

1-3) $(1+z)^{-m} = 1 - mz + \frac{m(m+1)z^2}{2!} - \frac{m(m+1)(m+2)z^3}{3!} + ...\ ...,\quad |z| < 1$

2) $\operatorname{sen} z = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{z^{2n-1}}{(2n-1)!},\quad |z| < \infty$

3) $\cos z = 1 + \sum_{n=1}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!},\quad |z| < \infty$ **;** **4)** $e^z = 1 + \sum_{n=1}^{\infty} \frac{z^n}{n!}\ ,\quad |z| < \infty$

SERIE DE FOURIER

Serie exponencial o compleja de Fourier

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jnw_0 t} \qquad \text{donde} \qquad c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t).e^{-jnw_0 t}dt \qquad \text{y} \qquad w_0 = \frac{2\pi}{T}$$

Serie trigonométrica de Fourier

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(nw_0 t) + b_n \operatorname{sen}(nw_0 t)\right) \quad \text{con} \quad a_0 = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\,dt \;;$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos(nw_0 t)\,dt \qquad b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\operatorname{sen}(nw_0 t)\,dt$$

Si f(t) es par:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(nw_0 t)\right) \quad \text{con :} \quad a_0 = \frac{4}{T}\int_{0}^{\frac{T}{2}} f(t)\,dt \;;\; a_n = \frac{4}{T}\int_{0}^{\frac{T}{2}} f(t)\cos(nw_0 t)\,dt \;;\; b_n = 0$$

Si f(t) es impar:

$$f(t) = \sum_{n=1}^{\infty}\left(b_n \operatorname{sen}(nw_0 t)\right) \quad \text{con :} \quad a_0 = a_n = 0 \;;\; b_n = \frac{4}{T}\int_{0}^{\frac{T}{2}} f(t)\operatorname{sen}(nw_0 t)\,dt$$

Si f(t) tiene simetría de media onda contiene solamente armónicas impares

$$f(t) = \sum_{n=1}^{\infty}\left(a_{2n-1}\cos(2n-1)w_0 t + b_{2n-1}\operatorname{sen}(2n-1)w_0 t\right) \qquad \text{con} \quad a_0 = 0$$

$$a_{2n-1} = \frac{4}{T}\int_{0}^{\frac{T}{2}} f(t)\cos(2n-1)w_0 t\,dt \qquad b_{2n-1} = \frac{4}{T}\int_{0}^{\frac{T}{2}} f(t)\operatorname{sen}(2n-1)w_0 t\,dt$$

Identidad de Parseval

Si utilizamos la serie compleja de Fourier: $\displaystyle \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |f(t)|^2 dt = \sum_{n=-\infty}^{\infty} |c_n|^2$

Si utilizamos la serie trigonométrica de Fourier: $\displaystyle \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |f(t)|^2 dt = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$

IDENTIDADES TRIGONOMÉTRICAS

$\operatorname{sen}(x) = \operatorname{sen}(x+2\pi)$ $\quad \cos(x) = \cos(x+2\pi)$ $\quad \operatorname{tg}(x) = \operatorname{tg}(x+\pi)$

$\operatorname{sen}(-x) = \operatorname{sen}(x+\pi)$ $\quad \cos(-x) = -\cos(x+\pi)$

$\operatorname{tg}(-x) = -\operatorname{tg}(x)$ $\quad \operatorname{cotg}(-x) = -\operatorname{cotg}(x)$

$\operatorname{sen}(x) = \cos\left(\frac{\pi}{2} - x\right)$ $\quad \cos(x) = \operatorname{sen}\left(\frac{\pi}{2} - x\right)$ $\quad \operatorname{tg}(x) = \operatorname{cotg}\left(\frac{\pi}{2} - x\right)$

$$a\operatorname{sen}(x) = b\cos(x) = \sqrt{a^2+b^2}\operatorname{sen}\left(x + \operatorname{arctg}\frac{b}{a}\right)$$

$$\operatorname{sen}^2(x) + \cos^2(x) = 1$$

$$\operatorname{sen}(x \pm y) = \operatorname{sen}(x)\cos(y) \pm \cos(x)\operatorname{sen}(y)$$
$$\cos(x \pm y) = \cos(x)\cos(y) \mp \operatorname{sen}(x)\operatorname{sen}(y)$$
$$\operatorname{tg}(x \pm y) = \frac{\operatorname{tg}(x) \pm \operatorname{tg}(y)}{1 \mp \operatorname{tg}(x)\operatorname{tg}(y)}$$

Fórmula del ángulo doble			
$\operatorname{sen}2\theta = 2\operatorname{sen}\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$	$\cos 2\theta = \cos^2\theta - \operatorname{sen}^2\theta = 2\cos^2\theta - 1 = 1 - 2\operatorname{sen}^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$	$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$	$\cot 2\theta = \frac{\cot\theta - \tan\theta}{2}$
Fórmula el ángulo triple			
$\operatorname{sen}3\theta = 3\operatorname{sen}\theta - 4\operatorname{sen}^3\theta$	$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$	$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$	
Fórmula del ángulo medio			
$\operatorname{sen}\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$	$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$	$\tan\frac{\theta}{2} = \csc\theta - \cot\theta = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\operatorname{sen}\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\operatorname{sen}\theta}$	$\cot\frac{\theta}{2} = \csc\theta + \cot\theta$

Fórmula de Euler

$$e^{ix} = \cos(x) + i\operatorname{sen}(x)$$
$$e^{-ix} = \cos(x) - i\operatorname{sen}(x)$$

FUNCIONES HIPERBÓLICAS

$$\mathrm{Ch}(\alpha) = \tfrac{1}{2}\left(e^{\alpha} + e^{-\alpha}\right) = Ch(-\alpha)$$
$$\mathrm{Th}(\alpha) = Sh(\alpha)/Ch(\alpha)$$
$$\mathrm{Ch}(\alpha) + \mathrm{Sh}(\alpha) = e^{\alpha}$$

$$\mathrm{Sh}(\alpha) = \tfrac{1}{2}\left(e^{\alpha} - e^{-\alpha}\right) = -Sh(-\alpha)$$
$$\mathrm{Ch}(\alpha) = \mathrm{Ch}(\alpha)/Sh(\alpha)$$
$$\mathrm{Ch}(\alpha) - Sh(\alpha) = e^{-\alpha}$$

$$\mathrm{Ch}^2(\alpha) - Sh^2(\alpha) = 1$$
$$\mathrm{Sh}(\alpha \pm \beta) = Sh(\alpha)\mathrm{Ch}(\beta) \pm Sh(\beta)\mathrm{Ch}(\alpha)$$
$$\mathrm{Ch}(\alpha \pm \beta) = \mathrm{Ch}(\alpha)\mathrm{Ch}(\beta) \pm \mathrm{Sh}(\alpha)\mathrm{Sh}(\beta)$$
$$\mathrm{Th}(\alpha \pm \beta) = \frac{\mathrm{Th}(\alpha) \pm \mathrm{Th}(\beta)}{1 \pm \mathrm{Th}(\alpha)\mathrm{Th}(\beta)}$$

$$\mathrm{Sh}(\alpha)\mathrm{Sh}(\beta) = \tfrac{1}{2}\left[\mathrm{Ch}(\alpha+\beta) - \mathrm{Ch}(\alpha-\beta)\right]$$
$$\mathrm{Ch}(\alpha)\mathrm{Ch}(\beta) = \tfrac{1}{2}\left[\mathrm{Ch}(\alpha+\beta) + \mathrm{Ch}(\alpha-\beta)\right]$$
$$\mathrm{Sh}(\alpha)\mathrm{Ch}(\beta) = \tfrac{1}{2}\left[\mathrm{Sh}(\alpha+\beta) + \mathrm{Sh}(\alpha-\beta)\right]$$
$$\left(\mathrm{Ch}(\alpha) \pm \mathrm{Sh}(\alpha)\right)^n = \mathrm{Ch}(n\alpha) \pm \mathrm{Sh}(n\alpha)$$

$$\mathrm{Sh}(xi) = i \cdot \mathrm{sen}(x)$$
$$\mathrm{Ch}(xi) = \cos(x)$$
$$\mathrm{Th}(xi) = i \cdot \mathrm{tg}(x)$$

$$\mathrm{sen}(xi) = i \cdot \mathrm{Sh}(x)$$
$$\cos(xi) = \mathrm{Ch}(x)$$
$$\mathrm{tg}(xi) = i \cdot \mathrm{Th}(x)$$

$$\mathrm{sen}(x+iy) = \mathrm{sen}(x)\mathrm{Ch}(y) + i \cdot \cos(x)\mathrm{Sh}(y)$$
$$\cos(x+iy) = \cos(x)\mathrm{Ch}(y) - i \cdot \mathrm{sen}(x)\mathrm{Sh}(y)$$
$$\mathrm{arc\,sen}(xi) = i\ \mathrm{ArgSh}(x)$$
$$\mathrm{arc\,cos}(xi) = -i\ \mathrm{ArgCh}(xi)$$
$$\mathrm{arc\,tg}(xi) = i \cdot \mathrm{ArgTh}(x) = \tfrac{1}{2}\ln\frac{1+x}{1-x}$$