Transformadas de Laplace

Definición

\[ F(s) = L\{f\}(s) = \int_0^\infty e^{-st} f(t) \, dt \]

Potencias

<table>
<thead>
<tr>
<th>(f(t))</th>
<th>(F(s) = L{f}(s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{1}{s})</td>
</tr>
<tr>
<td>(t)</td>
<td>(\frac{1}{s^2})</td>
</tr>
<tr>
<td>(t^n)</td>
<td>(\frac{n!}{s^{n+1}}, n) entero y positivo</td>
</tr>
<tr>
<td>(t^{-1/2})</td>
<td>(\sqrt{\frac{\pi}{s}})</td>
</tr>
<tr>
<td>(t^{1/2})</td>
<td>(\frac{\sqrt{\pi}}{2s^{3/2}})</td>
</tr>
<tr>
<td>(t^\alpha)</td>
<td>(\frac{\Gamma(\alpha + 1)}{s^{\alpha + 1}}, \alpha &gt; -1)</td>
</tr>
</tbody>
</table>

Funciones trigonométricas

<table>
<thead>
<tr>
<th>(f(t))</th>
<th>(F(s) = L{f}(s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin kt)</td>
<td>(\frac{k}{s^2 + k^2})</td>
</tr>
<tr>
<td>(\cos kt)</td>
<td>(\frac{s}{s^2 + k^2})</td>
</tr>
<tr>
<td>(\sin^2 kt)</td>
<td>(\frac{2k^2}{s(s^2 + 4k^2)})</td>
</tr>
<tr>
<td>(\cos^2 kt)</td>
<td>(\frac{s^2 + 2k^2}{s(s^2 + 4k^2)})</td>
</tr>
<tr>
<td>(t \sin kt)</td>
<td>(\frac{2ks}{(s^2 + k^2)^2})</td>
</tr>
<tr>
<td>(t \cos kt)</td>
<td>(\frac{s^2 - k^2}{(s^2 + k^2)^2})</td>
</tr>
<tr>
<td>(\frac{2(1 - \cos kt)}{t})</td>
<td>(\frac{\ln s^2 + k^2}{s^2})</td>
</tr>
<tr>
<td>(\frac{\sin at}{t})</td>
<td>(\arctan \left(\frac{a}{s}\right))</td>
</tr>
<tr>
<td>$f(t)$</td>
<td>$F(s) = L{f}(s)$</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>$\sin kt + kt \cos kt$</td>
<td>$\frac{2k s^2}{(s^2 + k^2)^2}$</td>
</tr>
<tr>
<td>$\sin kt - kt \cos kt$</td>
<td>$\frac{2k^3}{(s^2 - k^2)^2}$</td>
</tr>
<tr>
<td>$1 - \cos kt$</td>
<td>$\frac{k^2}{s(s^2 + k^2)}$</td>
</tr>
<tr>
<td>$kt - \sin kt$</td>
<td>$\frac{k^3}{s^2(s^2 + k^2)}$</td>
</tr>
<tr>
<td>$\frac{a \sin bt - b \sin at}{ab(a^2 - b^2)}$</td>
<td>$\frac{1}{(s^2 + a^2)(s^2 + b^2)}$</td>
</tr>
<tr>
<td>$\frac{\cos bt - \cos at}{a^2 - b^2}$</td>
<td>$\frac{s}{(s^2 + a^2)(s^2 + b^2)}$</td>
</tr>
<tr>
<td>$\sin at \cos bt$</td>
<td>$\frac{1}{2} \arctan \frac{a + b}{s} + \frac{1}{2} \arctan \frac{a - b}{s}$</td>
</tr>
<tr>
<td>$\frac{t}{\sin \omega t - \omega t \cos \omega t}$</td>
<td>$\frac{\omega^2}{(s^2 + \omega^2)^2}$</td>
</tr>
</tbody>
</table>

**Funciones hiperbólicas**

<table>
<thead>
<tr>
<th>$f(t)$</th>
<th>$F(s) = L{f}(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sinh kt$</td>
<td>$\frac{k}{s^2 - k^2}$</td>
</tr>
<tr>
<td>$\cosh kt$</td>
<td>$\frac{s}{s^2 - k^2}$</td>
</tr>
<tr>
<td>$\sinh^2 kt$</td>
<td>$\frac{2k^2}{s(s^2 - 4k^2)}$</td>
</tr>
<tr>
<td>$\cosh^2 kt$</td>
<td>$\frac{s^2 - 2k^2}{s(s^2 - 4k^2)}$</td>
</tr>
<tr>
<td>$t \sinh kt$</td>
<td>$\frac{2ks}{(s^2 - k^2)^2}$</td>
</tr>
<tr>
<td>$t \cosh kt$</td>
<td>$\frac{(s^2 - k^2)^2}{s^2 + k^2}$</td>
</tr>
<tr>
<td>$\frac{2(1 - \cosh kt)}{t}$</td>
<td>$\ln \frac{s^2 - k^2}{s^2}$</td>
</tr>
</tbody>
</table>
### Funciones exponenciales

<table>
<thead>
<tr>
<th>$f(t)$</th>
<th>$F(s) = L{f}(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^{at}$</td>
<td>$\frac{1}{s-a}$</td>
</tr>
<tr>
<td>$te^{at}$</td>
<td>$\frac{1}{(s-a)^2}$</td>
</tr>
<tr>
<td>$t^n e^{at}$</td>
<td>$\frac{n!}{(s-a)^{n+1}}$, $n$ entero y positivo</td>
</tr>
<tr>
<td>$e^{bt} - e^{at}$</td>
<td>$\ln \frac{s-a}{s-b}$</td>
</tr>
<tr>
<td>$\frac{1}{t} e^{-a^2/4t}$</td>
<td>$e^{-a\sqrt{s}}$</td>
</tr>
<tr>
<td>$\frac{\sqrt{\pi}a}{2\sqrt{\pi} t^3} e^{-a^2/4t}$</td>
<td>$\sqrt{s}$</td>
</tr>
<tr>
<td>$\frac{e^{bt} - e^{at}}{t}$</td>
<td>$\ln \frac{s-a}{s-b}$</td>
</tr>
<tr>
<td>$a - b$</td>
<td>$\frac{1}{(s-a)(s-b)}$</td>
</tr>
<tr>
<td>$ae^{at} - be^{bt}$</td>
<td>$\frac{1}{s}$</td>
</tr>
<tr>
<td>$1 - e^{-at}$</td>
<td>$\frac{1}{(s-a)(s-b)}$</td>
</tr>
<tr>
<td>$\frac{a}{at - 1 + e^{-at}}$</td>
<td>$\frac{1}{(s+a)s}$</td>
</tr>
<tr>
<td>$\frac{a^2}{t - \frac{1}{a} + (t + \frac{2}{a}) e^{-at}}$</td>
<td>$\frac{1}{(s+a)^2s^2}$</td>
</tr>
</tbody>
</table>

### Funciones exponenciales con trigonométricas

<table>
<thead>
<tr>
<th>$f(t)$</th>
<th>$F(s) = L{f}(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^{at} \sin \omega t$</td>
<td>$\frac{\omega}{(s-a)^2+\omega^2}$</td>
</tr>
<tr>
<td>$e^{at} \cos \omega t$</td>
<td>$\frac{s-a}{(s-a)^2+\omega^2}$</td>
</tr>
<tr>
<td>$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin \omega_n \sqrt{1-\zeta^2} t$</td>
<td>$\frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$</td>
</tr>
<tr>
<td>$- \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin \left(\omega_n \sqrt{1-\zeta^2} t - \phi\right)$</td>
<td>$s$</td>
</tr>
<tr>
<td>$\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$</td>
<td>$s^2 + 2\zeta \omega_n s + \omega_n^2$</td>
</tr>
<tr>
<td>$1 - \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin \left(\omega_n \sqrt{1-\zeta^2} t - \phi\right)$</td>
<td>$\frac{\omega_n^2}{(s^2 + 2\zeta \omega_n s + \omega_n^2)s}$</td>
</tr>
<tr>
<td>$\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$</td>
<td></td>
</tr>
</tbody>
</table>

3
Funciones exponenciales con hiperbólicas

\[ f(t) \quad F(s) = L\{ft\}(s) \]

\[
\begin{align*}
 e^{at} \sinh kt & \quad \frac{k}{(s - a)^2 - k^2} \\
 e^{at} \cosh kt & \quad \frac{s - a}{(s - a)^2 - k^2}
\end{align*}
\]

Funciones trigonométricas con hiperbólicas

\[ f(t) \quad F(s) = L\{f\}(s) \]

\[
\begin{align*}
 \sin kt \sinh kt & \quad \frac{2k^2 s}{s^2 + 4k^4} \\
 \sin kt \cosh kt & \quad \frac{k (s^2 + 2k^2)}{s^4 + 4k^4} \\
 \cos kt \sinh kt & \quad \frac{k (s^2 - 2k^2)}{s^4 + 4k^4} \\
 \cos kt \cosh kt & \quad \frac{s^3}{s^4 + 4k^4}
\end{align*}
\]

Función de Dirac

\[ f(t) \quad F(s) = L\{f\}(s) \]

\[
\begin{align*}
 \delta(t) & \quad 1 \\
 \delta(t - t_0) & \quad e^{-st_0}
\end{align*}
\]

Funciones con escalón unitario

\[ f(t) \quad F(s) = L\{f\}(s) \]

\[
\begin{align*}
 e^{at} f(t) & \quad \hat{F}(s - a) \\
 f(t - a) u(t - a) & \quad e^{-as} F(s) \\
 u(t - a) & \quad \frac{e^{-as}}{s}
\end{align*}
\]

Propiedades

\[ f(t) \quad F(s) = L\{f\}(s) \]

\[
\begin{align*}
 e^{at} f(t) & \quad \hat{F}(s - a) \\
 f(t - a) u(t - a) & \quad e^{-as} F(s) \\
 f^{(n)}(t) & \quad s^n F(s) - s^{(n-1)} f(0) - \ldots - f^{(n-1)}(0) \\
 t^n f(t) & \quad (-1)^n \frac{d^n}{ds^n} F(s) \\
 \int_0^t f(\tau) g(t - \tau) d\tau & \quad F(s) G(s)
\end{align*}
\]